# Recent developments in amplitude reduction for HELAC 2-Loop

Based on work with Costas Papadopoulos, Giussepe Bevilacqua, Dhimiter Canko and Nikos Dokmetzoglou

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# Outline

- HELAC 2-Loop
- Amplitude reduction
- Current Amplitude reduction status

# HELAC 2-Loop

Costas Papadopoulos introduced HELAC
[Kanaki and Papadopoulos, 2000]. 3 steps:

- 1. Amplitude Construction (Described by Costas.P. and Giussepe B. in the previous talks)
- 2. Amplitude Reduction at 2-loops (Focus of my talk today)
- 3. Integrals (Nikos Dokmetzoglous') talk after me

# 2-Loop Amplitude reduction

Reduction is done at the *integrand* level.
A generic 2-loop integrand can be written using the following scalar product set:

$$\{p_i \cdot p_j, k_i \cdot k_j, k_i \cdot p_j, k_i \cdot \eta_j\}$$

as well as any masses inside the loops. For the rest of the discussion we will ignore the case of massive loops.
The integrand therefore has the general form

$$\mathcal{R} = \frac{\mathcal{N}}{\mathcal{D}} = \frac{\sum_a c_a (z_1^{(a)})^{\beta_1} \ldots (z_{n_a}^{(a)})^{\beta_N}}{D_1 \ldots D_{N_p}} \tag{1}$$

where the $z_i$ are any of the scalar products in the set.

# 2-Loop Amplitude reduction

The above can we written in a more reduced form:

$$\mathcal{R} = \frac{\mathcal{N}}{\mathcal{D}} = \sum_{m=0}^{N_p} \sum_{\sigma} \frac{\sum_a \bar{c}_a (\bar{z}_1^{(a)})^{\alpha_1} \dots (\bar{z}_{N_a}^{(a)})^{\alpha_N}}{D_{\sigma_1} \dots D_{\sigma_m}} \tag{2}$$

where now the $\bar{z}_i$ are only the scalar products which cannot be eliminated by being written as linear combinations of $D_i$, know as irreducible scalar products (ISPs) or the transverse $k_i \cdot \eta_j$ and $\sigma$ is any subset of $\{1, \dots, N_p\}$ with m elements.

Write the numerator of the integrand level amplitude as follows:

## Numerator Formula

$$\mathcal{N} = \sum_{m=0}^{N_p} \sum_{\sigma} \sum_{a} \bar{c}_a \prod_{i}^{N_{T+ISP}} (\bar{z}_i)^{\alpha_i} \prod_{j} D_j \tag{3}$$

where $N_{T+ISP}$ is the number of ISP and transverse scalar products and $j \neq \sigma_i$ for all $i$ [Bevilacqua et al., 2024]

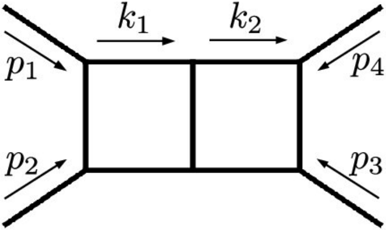
# 2-Loop Amplitude reduction

Our goal now is the calculate the coefficients $\bar{c}_a$ which generically depend on the set of scalar products. For each topology, we

- Identify the maximal set of loop propagators we can set to zero (Maximal Cut) and solve the equations that put all of them on shell simultaneously (cut equations)
- Write the equations of the coefficients $\mathbf{M} \cdot \vec{c} = \vec{\mathcal{N}}$ where $\mathbf{M}$ is a matrix of all monomials $\bar{z}_i$ evaluated on different values of cut-solutions, $\vec{c}$ is all the $c_a$ and $\vec{\mathcal{N}}$ is a vector of equal length with values of the numerator evaluated on the cut-solutions
- Solve the system of equations and move on to the next cut, where one less propagator is put on shell, AKA a subtopology.
- Do this till this for all subtopologies, and the reduction is complete

# 2-Loop reduction example

Let's look at a specific $2 \to 2$ topology example, part of current work:




Maximal cut: 7 propagators on shell. Question arises: Can/should we fit in 4 dimensions or $D = 4 - 2\epsilon$ dimension?

# 2-Loop reduction example: D-dimensions

In D-dimensions, begin with 11 free parameters: 8 from the components of 2 4-momenta, and 3 $\mu_{11}, \mu_{22}, \mu_{12}$ the $\epsilon$ components of $k_1^2$, $k_2^2$ and $k_1 \cdot k_2$ respectively.

With 7 cut equations, we have a remainder of 4 free parameters.

The right hand side of equation (3) (the Ansatz polynomial) has a total of 65 monomials, i.e. 65 coefficients to be fitted.

Use the 4 free parameters to get 65 sets of solutions in order to solve the system.

Challenge:Get a set of solutions to the cut equations which give an **M** matrix of rank 65.

Success! We have completed a Mathematica simulation of this fit.

How to do this numerically? -Hopeful progress (see the previous talk by Giuseppe B.)

# 2-Loop reduction example: 4-dimensions

In 4-dimensions, we begin with 8 free parameters which we can use to construct solutions to the cut equations, so after imposing the on-shell condition only 1 parameter left to build solutions with.

Problem!Cut solution sets with 1 free parameter cannot generate a matrix of rank of rank 65!

(Partial?) Success In 4-dimensions, we can use Gram determinant relations to reduce the number of coefficient we need to fit [Badger et al., 2012]. We find 24 for the example of the $2 \to 2$ double-box.

Completed a Mathematica simulation for the maximal cut.

# 2-Loop reduction example: 4-dimensions

But [Frellesvig, 2014] mention issues with lower topology cuts:

- A) Some sub-topologies having divergences that require careful subtraction
- B) Other sub-topologies where $\mathrm{Rank}[\mathbf{M}] < \mathrm{Length}[\vec{c}]$, i.e not enough independent equations to fit the polynomial Ansatz.

Both currently under investigation with our approach

# Next Steps/Outlook

Immediate steps

- Determine if a 4-d fit is possible and determine for which cases and why it may not be
- Implement the D-dimensional fit at one loop

Outlook

- D-dimensional fit at 2-loops for all $2 \to 2$ topologies
- $2 \to 3$ topology reduction

# Thank you for listening!

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